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II.

NOTE ON THE DEPENDENCE OF VISCOSITY ON PRESSURE AND TEMPERATURE.

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1. *Historical.*— In the following paragraphs, I endeavor to give a preliminary account of what may be called the isothermals, the isopiestics, and the isometrics with respect to viscosity. Notwithstanding the great geological importance* of these relations, nobody has as yet attempted to represent them systematically.

2. *The Material chosen.*— In order to obtain pronounced results for the effect of pressure on viscosity, substances must be selected on which temperature has a similarly obvious effect. For, in addition to the direct access to the molecule which is beyond the reach of pressure, temperature has the same marked influence on the expansion mechanism per unit of volume increment as the other agency. Hence liquids like marine glue, pitch, etc., which change continuously from solid to liquid, and in which this change takes place at an enormously rapid rate and is complete within relatively few degrees, are especially available for the present investigation.

The following data refer to *marine glue.* Viscosity is considered as a physical quality, and apart from such chemical considerations as are introduced in passing from one body to another.

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* This will be indicated by Mr. Clarence King, for whom this work was done.

3. *Definitions.*— In my paper[*] on the absolute viscosity of the three states of aggregation, I defined a fluid (liquid or gas) as a body which, under constant conditions of pressure, temperature, and strain, shows constant viscosity as to time. In a solid, *cœteris paribus,* viscosity markedly increases with the time during which stress is brought to bear. The molecular instabilities of a liquid, therefore, are supplied at the same rate in which they are used in promoting viscous motion. In a solid they are used more rapidly than the small rate of continuous supply.

The point of essential concern in these definitions is the constancy of stress, and its value below a certain critical datum. For instance, if in a solid stress be *increased* at the (small) rate necessary to insure a constant supply of instabilities, then solid viscosity will also be constant, and I am by no means sure that in such a case[†] yield points would eventually present themselves as breaks in the continuity of the solid view.

On the other hand stress may be conceived to increase so fast, that even a liquid fails to present sufficient instabilities for truly viscous motion. The elasticity and brittleness of many viscous liquids, especially at low temperatures, is a case in point.

4. *Hardness.* — Throughout my work on viscosity,[‡] I have adverted to the association of viscosity with zero forces acting for infinite times, and of hardness with infinite forces (relatively) acting for zero times, and have adduced many new examples showing the utter distinctiveness of these two properties. The subject of hardness has, however, recently taken more definite shape in the researches of Auerbach,[§] based on a principle due to Hertz.[||] According to these observers, hardness is an expression for the elastic limits of a body in case of contact between its plane surface and the curved surface of some other (harder) body. Hardness so defined admits of absolute measurement in terms of dynes per square centimeter.

5. *Method of Work.*— In all experiments like the present, one cannot be too careful to preconsider the conditions under which the results are obtained; for one is only too apt to attribute an absence

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[*] Phil. Mag. (5), Vol. XXIX. p. 337, 1890. Cf. p. 354.

[†] Cf. Am. Journal (3), Vol. XXXIV. p. 19, 1887.

[‡] Phil. Mag. (5), Vol. XXVI. p. 210, 1888. Cf. Bull. U. S. Geolog. Survey, No 73, pp. 42–44, 97, 98, 1891.

[§] Auerbach, Wied. Ann., Vol. XLIII. p. 61, 1891.

[||] Hertz, Crelle's Journal, Vol. XCVI. p. 156, 1882.

of flow to the effect of pressure on viscosity, when the real cause is to be found in the geometry of the apparatus employed. I have therefore availed myself of transpiration methods, since the theory of the experiments is in this case very fully given.*

The marine glue, § 2, was forced out of a sufficiently large reservoir, through tubes of steel about 10 cm. long, and 0.5 to 1 cm. in diameter. Pressures as high as 2,000 atm. were applied at the reservoir, by aid of my screw compressor.† Temperatures between 10° and 30° were kept constant by a suitable water bath. Throughout the work the flow was so excessively slow (amounting to an advance of only a few millimeters per hour), that Poiseuille's law was at once applicable. The only considerable source of error in the work is the occurrence of more or less incidental slipping. However, inasmuch as the outflow of marine glue is capped by a rounded surface, it follows that the flow is most marked at the axis of the tube compatibly with the theory of the experiment.

6. *Volume Viscosity.*— At the end of stated intervals of time (usually hours), the small cylinders of marine glue which had exuded were cut off with a sharp knife, and weighed. Now it was curious to note that these cylinders, left to themselves for about a day, showed a gradual and marked deformation, such that the originally plane bottom or surface of section eventually expanded into a symmetrical projecting conoid, with an acute apex angle of less than 45°. I take this to be an example of volume viscosity. The restitution of volume is greatest in the axis of the cylinder where the flow is a maximum, and where the matter has been crowded into the smallest space. As a whole, the experiment is somewhat puzzling, for it points to the occurrence of a notable amount of slowly reacting elasticity even in this truly viscous solid. Indeed, as time went on, a re-entrant conoid, corresponding to the projecting cone just described, gradually dimpled the second of the two surfaces of section. What is here indicated, therefore, is probably a surface of flow.

7. *Viscosity and Pressure. Isotherms.* — Table I. gives a summary of my chief results. The table is one of double entry, and the data contained show the absolute viscosity (η) of marine glue at the stated temperatures and pressures, in terms of one billion

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* In addition to the well known work of Poiseuille, cf. Hagenbach, Pogg. Ann., Vol. CIX. p. 385, 1860; Osborne Reynolds, Phil. Trans., Vol. III., 1883, p. 935; Wilberforce, Phil. Mag. (5), Vol. XXXI. p. 407, 1891.

† These Proceedings, Vol. XXV. p. 93, 1890.

g/cs units. The pressure excess is the difference of pressures at the two ends of the tube.

TABLE I. — MEAN VALUES OF $\eta/10^9$ FOR MARINE GLUE.

Δp = Pressure Excess =	100 atm.	300 atm.	1000 atm.	1500 atm.	2000 atm.
Temperature = 8.5°	—	—	$> 30\,000$*	—	$> 60\,000$*
" = 18.3°	(2.5)	—	8.30	12.0	15.2
" = 22.5°	—	—	1.12	—	2.2
" = 30.5°	.065	.073	—	—	—
Rates at	8.5°	18.3°	22.5°	30.5°	
$10^{-9} \times \Delta\eta / \frac{1}{2}\Delta p =$	—	.0137	.00220		
$\alpha = \Delta\eta / \eta_0 \times \frac{1}{2}\Delta p =$	—	.0091	.020		

In constructing the rate of change of viscosity with pressure, I assumed that the whole thread *transpired at the mean pressure at the two ends of the steel tube*; or since the pressure at the open end is zero, at half the pressure excess. Furthermore, that

$$\eta_p = \eta_0 (1 + \alpha p) = \eta_0 (1 + \tfrac{1}{2} \alpha \Delta p).$$

If therefore $\Delta\ \eta$ be the increment of viscosity corresponding to the pressure $\frac{1}{2}\ \Delta\ p$, the final data of Table I. (rates) are at once intelligible.

In view of the occurrence of the arbitrary variable η_0, it is hardly probable that α would be independent of temperature. If reference were made to some other pressure, viz. $\alpha' = (\eta_p - \eta_0) / \eta_P$, the values of α' approach each other in proportion as P is larger. Thus, for $P = 1000$ atm., $\alpha' = .009$ at 18°.3 and $\alpha' = .010$ at 22°.5. The point of immediate importance here, however, is that *in proportion as the viscosity of a body increases with fall of temperature, its isothermal rate of increase with pressure also increases ;* and that the rate of increase in the latter instance is a coefficient as remarkably large as $\alpha = .01$ to $.02$.

8. *Viscosity and Temperature. Isopiestics.*— Table I. also enables me to construct the approximate isopiestics for $\Delta\ p = 0$, 500,

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* No flow perceptible after five hours, in case of a tube 10 cm. long and 1 cm. in diameter.

1000, and 2000 atm. These loci are distinctly curved, showing that viscosity decreases at an accelerated rate with rise of temperature.

9. *Conditions of Constant Viscosity. Isometrics.*— In order, however, to compare the relative effects of temperature and pressure, it is necessary to map out the lines of equal viscosity for marine glue. I endeavored to do this by constructing Table I. graphically; thus I attained the data of Table II., where Δp is the pressure excess corresponding to the temperature θ, for the case in which the viscosity η has the value given at the head of the table.

TABLE II. — ISOMETRICS AS REGARDS VISCOSITY FOR MARINE GLUE.

$\eta = 1 \times 10^9$		$\eta = 2 \times 10^9$		$\eta = 3 \times 10^9$		$\eta = 5 \times 10^9$	
Δp	θ	Δp	θ	Δp	θ	Δp	θ
atm.	° C.	atm.	° C.	atm.	° C.	atm.	° C.
0	19.1	0	17.7	0	16.8	0	—
500	21.7	500	20.4	500	19.6	500	18.3
1000	22.8	1000	21.4	1000	20.6	1000	19.4
2000	24.3	2000	22.7	2000	21.8	2000	20.5
$\beta = .0030$		$\beta = .0026$		$\beta = .0024$		$\beta = .0022$	

There are thus given a family of lines of decreasing curvature. Supposing them to be eventually more nearly linear I have constructed $\beta = \Delta \theta / \frac{1}{2} \Delta p$ (where $\Delta \theta$ and $\frac{1}{2} \Delta p$ are the corresponding increments of temperature and pressure for constant viscosity) for the region between $\Delta p = 1000$ and 2000 atm. See Table II. These values, crude as they are, have an important geological bearing on the fluidity of magmas. I infer that in a field of high pressure as small a rise of temperature as 1° C. will quite wipe out the increment of viscosity due to a pressure as large as 400 atmospheres.

10. *Isothermal Flow through Tubes.*—Let there be given a tube of length l and radius ρ. Let $\eta = \eta_0 (1 + \alpha p)$ be the viscosity of the viscous liquid forced through it by the pressure excess $\Delta p = 2p$ (so that there is no pressure at one end of the tube) and at the constant temperature θ. The length λ of the cylinder of fluid issuing per unit of time (t) will be

$$\frac{\lambda}{t} = \frac{\Delta p \rho^2}{8 l \eta_0 (1 + \frac{1}{2} a \Delta p)}.$$

Hence the maximum flow will occur when $\Delta p = -2 / a$, or, as negative pressures are excluded, the function λ / t is of a kind continually increasing with Δp.

11. *Measurement of excessively high Pressures.*— In view of this property of λ / t, it is worth inquiring in how far the transpiration method is available for high pressure measurement, when all other means fail.

Take for example marine glue, and a tube $\frac{1}{16}$ inch in diameter and 1 inch long. Then the mass m of marine glue which at 18°.3 would exude per hour is given in Table III.

TABLE III. — TRANSPIRATION OF MARINE GLUE, PER HOUR, AT 18°.3 C.

$l = 2.5$ cm. $\rho = .08$ cm. $\eta = 1.5 (1 + .009\frac{1}{2} \Delta p) \times 10^9$.

Δp	=	1000,	5000,	10000,	15000,	20000	atm.
$m \times 10^6$	=	4232,	4949,	5060,	5096,	5115	grams.

Thus it appears that whereas a hole $\frac{1}{16}$ inch in diameter may be efficiently sealed by marine glue at 18°.3 C., pressure measurement by aid of the exuding mass is impossible above 10,000 atm., whereas even between 5,000 atm. and 10,000 atm. the method is insensitive. To use a method like the present for very high pressure measurement, a substance of smaller pressure coefficient must therefore be sought, if such a one with other necessary qualities, exists. It is with the object of searching for such a body, as well as of finding the maximum of hydrostatic pressure attainable in the laboratory, that I had a tinned screw and socket constructed,* and hope to be able to report the results of my work at an early opportunity.

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* These Proceedings, Vol. XXV. pp. 94, 108, 1890.

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